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Support for the Reality of Quarks

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There are still some doubts about the existence of quarks among a minority of physicists who believe that they could merely represent a symmetry without having a physical reality. This is because a rigorous element of reality for quarks can only correspond to free observed quarks. An independent argument in favor of the existence of quarks as real particles is given using a method based on Mayer's cluster expansion to calculate the critical temperature for a phase transition of a nonideal quark–antiquark plasma to a hadron fluid.

1. INTRODUCTION

Although the theory of QCD has made considerable effort to describe the world of the strong interaction, a rigorous proof which could show without doubt the existence of quarks as real existing particles is lacking. The analysis of jet experiments merely shows that QCD is correct and that the observed events are sensitive to the color structure of quarks, but does not rigorously prove the existence of quarks as real objective particles. Quarks are confined inside the hadrons and cannot be observed individually. The only hints which we have are (1) the parton model, which shows that there must be pointlike particles inside the hadrons, and (2) the non-Abelian symmetries of QCD and the corresponding perturbative field theory. But regarding the first hint, we realize that the parton model itself does not give us a closed and rigorous picture of the properties of these particles as such. Perhaps, in the framework of this model, if we want to be cautious, the particles inside the hadrons should still be called partons instead of quarks. Regarding the second hint, one could still assert that quarks are merely a representation or artifact of the non-Abelian symmetries and are

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not objects having an independent reality like particles or waves, and in particular that the confinement problem is still unsolved to show us that these objects indeed carry color charge. “The question of whether quarks, gluons and colour are to be regarded as elements of reality has to be decided on the basis of the observables, that is, without recourse to gauge fields” (Buchholz, 1996; see also Haag, 1993). Buchholz recently has shown on the basis of the Schwinger model that confinement is not necessarily a result of gauge symmetries (Buchholz, 1996). So, one probably can conclude that gauge symmetries are not a sufficient condition for confinement. Or, in the words of Seiler, “Even though we often talk about quarks and gluons as ‘particles’ we do not really mean it: The particles described by QCD are supposed to be only hadrons” (Seiler, 1985).

Under such conditions, alternative methods that could confirm the existence of quarks as particles become essential. One of these methods can be found in plasma physics. The advantage of plasma physical methods is that we do not have to investigate one separate quark as, for example, particle physicists do in scattering processes in a Wilson chamber. This cannot be realized, since quarks are confined. But in a plasma, in principle, quarks can be observed as free particles, because of asymptotic freedom. The situation is similar in the well-investigated electron-ion plasma. From the interaction forces between the particles there, via statistical and kinetic methods, predictions can be made about the collective behavior of the plasma. Conversely, from the collective behavior of the plasma, conclusions can be made about the features of the particles inside the plasma.

The aim of the present paper is the following: using Mayer’s cluster expansion method, we calculate the temperature where a nonideal quark–antiquark plasma condenses into droplets of quark–antiquark pairs, i.e., into a fluid of mesons. The process is similar to the formation of morning dew from fog in the late night hours when the temperature falls. But the method applied here is different than the one used to investigate droplet formation of nucleons in heavy ion collisions. In the latter method, the probability of droplet formation is estimated by calculating the change in the Gibbs free energy of the system when a droplet appears, while in the former method particles of the plasma cluster to droplets, i.e., the mesons *are* the droplets. The method can be considered as a method to estimate the critical temperature analytically on a heuristic basis independent of other models.

The obtained temperature where this process takes place is then compared to the critical temperature calculated by conventional methods, like finite-temperature QCD phase transitions (for a recent review of which see Meyer-Ortmanns, 1996). If they agree, we have as an additional result another independent confirmation in favor of the fact that the particle interpretation of quarks can be entered in QCD through the use of Feynman rules for the gauge and matter fields. This conclusion can be made on the basis that the cluster expansion method is independent from the latter method and considers distinguished particles, like, for example, the electrons in an ordinary ionized gas, which are treated classically there. Since a quark–gluon plasma exists only under the condition of extreme high temperature, the neglect of quantum mechanical corrections is justified and the quarks can be considered as classical particles. In particular, no assumptions about nonobservable statements are made.

We apply this method to a plasma of heavy quarks, i.e., charms and bottom, and consider the phase transition to J/ψ and Υ mesons, respectively. In these cases a nonrelativistic investigation will suffice. Considering only heavy quarks and neglecting the thermodynamics of light quarks is justified by the fact that light and heavy particles have different temperatures in a plasma because the great difference between the masses make the exchange of energy between the two kinds of particles difficult, as is well known from conventional plasma physics (Lifshitz and Pitaevski, 1981). In a two-component plasma consisting of a heavy and a light component, each component can be considered in equilibrium separately although the plasma as a whole is in a nonequilibrium state. This is because the relaxation time for the whole plasma is much longer than the relaxation times for each component. Investigating the formation of quark–antiquark clusters is also supported by the existence of diquark correlations in a quark–gluon plasma (Anselmino *et al.*, 1993). Generally, it is believed that the late evolution of the quark–gluon plasma is its hadronization at a critical temperature T_c. It should be noticed here that in an actual QCD plasma the transition is driven by gluons and light quarks. Heavy quarks are just external probes that feel the change in the static potential. That is why charmonium dissociation is a thermometer (Karsch *et al.*, 1988). Hence the results obtained here are just estimations representing a qualitative picture only. There are many papers using different approaches based on lattice QCD, finite-temperature quantum field theory, and phenomenological methods to calculate T_c. The lowest value for T_c for a phase transition in QCD known to the author from the literature is $T_c = 103 \pm 32$ MeV (Bonometto and Patano, 1993), where the uncertainty is due to casual errors, and the largest value is $T_c = 260$ MeV (Boyd *et al.*, 1996). The latter result is due to pure gauge theory. Recently, more exact results for the phase transition of a light quark plasma to hadrons appeared in the correponding literature. But here, we are considering a hypothetical plasma made of heavy quarks and since we are in search of qualitative results only, we take the first of the above results as a lower and the second as an upper bound for T_c. Hence, if our hypothesis is correct, we expect that the results obtained in this paper to lie within this range.

2. DROPLET FORMATION BY CLUSTERING OF PARTICLES

To calculate the condensation temperature for clustering, we start with a brief presentation of the mechanism used here for droplet formation by clustering of particles: From Mayer's cluster expansion method, we know that for a nonideal gas the number of particles N and the pressure P are

$$N = \sum_{l=1}^{\infty} \frac{lVb_l}{\lambda^{3l}} exp\left\{\frac{\mu l}{T}\right\} \tag{1}$$

$$P = T \sum_{l=1}^{\infty} \frac{b_l}{\lambda^{3l}} \exp\left\{\frac{\mu l}{T}\right\} \tag{2}$$

where T, V, and μ are the temperature, volume, and chemical potential, respectively, $\lambda = \sqrt{2\pi/mT}$, m is the mass of the particles, and b_l are the cluster integrals

$$b_l = \frac{1}{l!V} \int_{-\infty}^{+\infty} \left(\sum_{1,\ldots,l}^{\bowtie} \prod f_{ik} \right) d\vec{r}_1 \ldots \vec{r}_l, \qquad l = 1, 2, \ldots, \infty \tag{3}$$

where $\Sigma^{\bowtie}$ means the summation over a connected complex and we define here

$$f_{ij} = 1 - \exp\{\beta U(r_{ij})\} \tag{4}$$

where $r_{ij} = |\vec{r}_i - \vec{r}_j|$, i, $j = 1, 2, \ldots, N$, is the distance between particles i and j, U is the potential, and $\beta = 1/T$. Hence, b_l connects l particles.

Defining

$$m_l = \frac{Vb_l}{\lambda^{3l}} \exp\left\{\frac{\mu_l}{T}\right\} \tag{5}$$

we obtain from (1)

$$N = \sum_{l=1}^{\infty} lm_l \tag{6}$$

m_l is the number of droplets, where each droplet contains l elementary particles. From (2) we obtain

$$PV = N'T \qquad \text{with} \quad N' = \sum_l m_l \tag{7}$$

Equation (7) has the form of an equation of state for an ideal gas. N' is the total number of droplets. Hence, the initial nonideal gas (the plasma) has been phase-transformed to an ideal gas of droplets, where the latter can be considered as a fluid in the sense of gas dynamic theory. This is the mechanism we apply here.

3. QUARKONIUM AS CLUSTERS OF QUARK–ANTIQUARKS AND T_c

Since the formation of diquarks is energetically favorable for two quarks in a spin-zero configuration, the deconfined quarks pair up in the plasma (Ansemino *et al.*, 1993, Sec. 5.D). It was also shown that weak nonleptonic decays of ordinary quark–antiquark mesons are mediated by color-triplet diquark virtual states (Anselmino *et al.*, 1993, Sec. 3.C). These arguments confirm the assumption that quarks and antiquarks cluster together as quark–antiquark pairs when the plasma temperature falls below the deconfining point.

We consider a plasma consisting of charm–anticharm quarks, and as a further investigation another plasma consisting of bottom–antibottom quarks. We consider the phase transitions $c\bar{c} \rightarrow J/\psi$ and $b\bar{b} \rightarrow \Upsilon$, respectively. It is well known that the binding potential at zero temperature is given by the one-gluon exchange contribution plus the linear confining part

$$V(r, 0) = -\frac{\alpha}{r} + \sigma r \tag{8}$$

where α is the coupling constant and σ the string tension coefficient. This is the so-called Cornell potential (Eichten *et al.*, 1978, 1980), which in the temperature environment is color-screened (Joos and Montvay, 1983) [we use here the representation of $U(r)$ from Karsch *et al.*, (1988)]:

$$U(r) = \frac{\sigma}{\omega}(1 - e^{-\omega r}) - \frac{\alpha}{r} e^{-\omega r} \tag{9}$$

where ω is the Debye screening length. We neglect spin forces as a first approximation and assume that initially (for $T > T_c$) we have a homogeneous nonideal quark–antiquark plasma without boundary. Hence, the total number of quarks and antiquarks is $N \rightarrow \infty$ and the volume $V \rightarrow \infty$. The chemical potential μ of the quarks and antiquarks is to be taken zero, since in this method after the phase transition the mesons are considered as droplets of quark–antiquarks and hence there is no change in the particle number of the quarks and antiquarks. From (5) follows then for the number of the droplets of the quark–antiquark pairs

$$m_2 = \frac{V b_2}{\lambda^6} \tag{10}$$

Since we consider only the transition of a quark–antiquark plasma to mesons, we have $m_l = 0$ for $l \neq 2$. Hence, we can write $m_2 = N/2$. Defining $n = N/V$ as the number density of the particles in the plasma at the moment where the phase transition takes place, we obtain from (10) for the temperature at the moment of the droplet formation

$$T_c = \frac{2\pi}{m}\left(\frac{n}{2b_2}\right)^{1/3} \tag{11}$$

To calculate b_2, first we separate from the potential $U(r)$ in (9) the constant term σ/ω. This will change no thermodynamic quantities, since they are proportional to derivatives of the logarithm of the partition function. Hence, constant terms in the potential factorize and cancel. Then, we obtain from (9) and (3) for the quark–antiquark pairs and for $V \to \infty$

$$b_2 = 2\pi \int_0^\infty \left[1 - \exp\left\{-\beta\left(\frac{\sigma}{\omega} + \frac{\alpha}{r}\right)e^{-\omega r}\right\}\right] r^2\, dr \tag{12}$$

Since the integrand is zero at the two limits $r \to 0$ and $r \to \infty$, the integral is convergent. We estimate this integral by some simple approximations: the integral depends strongly on the behavior of the exponential function in the exponent. We have

$$\beta\left(\frac{\sigma}{\omega} + \frac{\alpha}{r}\right)e^{-\omega r} \gg 1 \qquad \text{for} \quad \omega r \ll 1 \tag{13}$$

$$\beta\left(\frac{\sigma}{\omega} + \frac{\alpha}{r}\right)e^{-\omega r} \ll 1 \qquad \text{for} \quad \omega r \gg 1 \tag{14}$$

Hence, as an approximation, we can write

$$b_2 \approx 2\pi\left[\int_0^{1/\omega} r^2\, dr + \int_{1/\omega}^\infty \beta\left(\frac{\sigma}{\omega} + \frac{\alpha}{r}\right)e^{-\omega r}\, r^2\, dr\right]. \tag{15}$$

The solutions of these integrals are tabulated in Gradshteyn and Ryshik (1994). We obtain

$$b_2 = 2\pi\left[\frac{1}{3\omega^3} + \frac{\beta}{e}\left(\frac{5\sigma}{\omega^4} + \frac{2\alpha}{\omega^2}\right)\right] \tag{16}$$

Since the plasma is assumed to be homogeneous, we can define an R as the distance between two neighboring particles. Then the density of the plasma becomes $n = 1/(4/3)\pi(R/2)^3$. If R at the moment of or shortly before the phase transition is designated by R_c, we obtain from (11) and (16) a cubic equation for T_c,

$$T_c^3 + \frac{3}{e}\left(\frac{5\sigma}{\omega} + 2\alpha\omega\right)T_c^2 - \frac{36\pi\omega^3}{m^3 R_c^3} = 0 \tag{17}$$

To solve this equation for T_c, we first have to fix the parameters. We have

$\sigma = 0.192$ GeV^2 and $\alpha = 0.471$ (Jacobs *et al.*, 1986). It is known that for $T \leq 1.5T_c$ the spatial string tension can be considered as temperature independent (Gubankova and Simonow, 1995). In the critical phase ω was calculated in Karsch *et al.* (1988). For J/ψ, $\omega_c = 0.699$ GeV and for Υ, $\omega_c = 1.565$ GeV. The masses of the charm and bottom quarks are $m = 1.320$ GeV and $m = 4.746$ GeV, respectively. To calculate (15), it remains to fix R_c or equivalently the density of the quark–antiquark plasma in or shortly before the critical phase. The dissociation radius for J/ψ and Υ is also calculated in Karsch *et al.* (1988). There, semiclassical calculations give $R_c = 0.87$ fm for J/ψ and $R_c = 0.33$ fm for Υ, whereas quantum mechanical calculations lead to diverging radii when $\omega \to \omega_c$. Hence, concerning R_c, it is not clear how to use these results. But, since we are approaching the critical point from the plasma phase to the hadronized phase, we can apply the following two simple arguments: (1) To form hadrons, the quarks must approach each other within the range of the size of the hadrons. The typical size of a hadron is about 1 fm, which is also the range of the strong force. (2) After the droplets are formed, i.e., hadronization is completed, the hadron gas must be free [in the model represented here, see Eq. (7)]; hence the hadrons cannot come closer to each other than the range of the strong force, but at the same time cannot be further from each other than this range, because then the homogenity of the plasma shortly before the transition would break down. This picture is also confirmed by the fact that the plasma phase region is the region where $P \neq 0$, and when the plasma condenses to nuclear matter at ordinary density the pressure becomes $P = 0$ (Chaplin and Nauenberger, 1977). Nuclear matter in this case consists of matter where the nucleons lie side by side, like balls poured into a vessel which is under a condition of weightlessness. Hence, as a first approximation, we can set $R_c = 1$ fm. With this value, it follows from (17) that $T_c = 228$ MeV for the formation of J/ψ and $T_c = 111$ MeV for the formation of Υ (all other solutions are negative). We see that both values lie within the range of 100–260 MeV, where a phase transition generally is expected.

A question that should be clarified is the difference between the critical temperatures of J/ψ and Υ formation. Although both temperatures lie within the range of the critical region, they differ by a factor two. Probably this is in relation to the confinement radii of charm and bottom quarks, which can be different. As mentioned previously, semiclassical calculations in Karsch *et al.* (1988) result in a smaller dissociation radius for Υ than for J/ψ. From (11) it follows that the critical temperature increases when the dissociation radius decreases, but is inversely proportional to the mass of the particles. Hence, the critical temperatures could be assimilated. On the other hand, physically, a lower critical temperature for the heavier particles is expected, since the temperature of heavier particles in a nonequilibrium plasma usually is lower than that for lighter particles before equilibrium is reached. The reason is that lighter particles initially absorb energy faster than heavier particles because of inertia. Here, the dissociation radii were roughly approximated by general principles. More exact radii of course would result in more exact values for the critical temperatures. But since both T_c lie within the range of the temperature where a phase transition can be expected in general, the hypothesis given at the beginning of the paper is fulfilled.

4. CONCLUSIONS

We have studied a nonideal quark–antiquark plasma consisting of charm or bottom, which are heavy enough for the nonrelativistic approximation to be applied. The plasma is assumed to be a classical gas of individual particles with a "Coulomb" plus a confining potential. The assumption of a classical gas can be made here because quantum statistics can be neglected in the hadronic gas phase for $T_c \geq 50$ MeV (Rafelski, 1982, Sec. 6.2). No quantum field-theoretic symmetries or methods are applied. The obtained critical temperatures lie within the range of results generally obtained from QCD. In consideration of the classical nature of the method, the results obtained here agree with the assumption that quarks are real existing particles. In other words, we can conclude the following picture: At high temperature there can exist a nonrelativistic plasma of heavy particles and antiparticles (which is a classical gas due to the high temperature). This plasma can condense into heavy mesons at a critical temperature. The particles are called quarks because (1) they obey a confining potential and (2) the results are consistent with QCD.

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